Accuracy Criteria for Evaluating Supersonic Missile Aerodynamic Coefficient Predictions

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Aerodynamic prediction methods are traditionally compared with wind-tunnel test data. However, the assessment of accuracy is left to an arbitrary interpretation. An accuracy criterion has been developed that defines the required prediction accuracy in terms of allowable errors in missile performance and missile design parameters. Equations have been selected that relate these parameters to the aerodynamic drag, stability, and control coefficients. These equations are differentiated with respect to the aerodynamic coefficients and simplified when possible. Allowable errors in the performance or design parameters are estimated, based on preliminary design requirements, and the required aerodynamic coefficient accuracy calculated. The results allow a quantitative evaluation of prediction accuracy.

Nomenclature

A	= wing or fin area
b	= reference length
C_A	= axial force coefficient
C_D	= drag coefficient
C_L	= lift coefficient
C_l	= rolling moment coefficient
C_m	= pitching moment coefficient
C_N	= normal force coefficient
C_n	= yawing moment coefficient
C_t	= thrust coefficient
C_Y	= side force coefficient
g	= gravitational constant
h	= altitude
I	= moment of inertia
K	= stability parameter
K_S	= longitudinal static stability parameter
K_{S1}	= bank/roll static stability parameter
K_{S3}	= yaw static stability parameter
K_X	= roll-yaw cross-coupling parameter
k	= induced drag factor
k_{DV}	= slope of $C_{D_0} V^2$ vs V curve
N/A	= not applicable
n	= aerodynamic load factor
P_S	= specific excess power
q	= dynamic pressure
R	= range
r	= turn radius
S	= reference area
T	= thrust
V	= velocity
V_B	= velocity at which linear $C_{D_0} V^2$ curve intercepts axis
$\dot{V}$	= acceleration
W	= weight
X_{CG}	= axial center of gravity
X_{CP}	= axial center of pressure
Y_{CP}	= wing lateral moment arm about centerline
α	= angle of attack
β	= yaw angle
β_a	= atmosphere density exponent
δ	= control deflection
δA	= aileron deflection
δR	= rudder deflection
Δ	= parameter increment
Γ	= dihedral angle
ρ	= atmospheric density
ρ_{ref}	= reference density
ϕ	= bank angle
$\dot{\psi}$	= angular turn rate
τ	= response time

Superscript

$(\ )'$	= refereced to panel area

Subscripts

B	= body alone
CONTROL	= control value
F	= final value
T	= tail value
TRIM	= trim value
W	= wing value
α	= derivative with α
β	= derivatie with β
δ	= derivative with δ
δA	= derivative with δA
δR	= derivative with δR
0	= initial value for V, W; also value at $\alpha = 0$ deg for C_A and C_D

Introduction

RESULTS from aerodynamic prediction methdos are constantly being compared with wind-tunnel data. However, in the majority of cases, the assessment of accuracy is left to the viewer's interpretation of what is a good or poor comparison. Figure 1 is a typical example of a pitching moment comparison. Is it a good or poor prediction of C_m? The purpose of this paper is to provide accuracy criteria for supersonic missiles that answer this question for the six static aerodynamic force and moment coefficients, C_A, C_N, C_m, C_Y, C_n, and C_l. The paper presents the selection of governing equations, development of accuracy equations, selection of allowable performance/design errors, and examples of allowable coefficient accuracies.

Two primary reasons aerodynamic coefficients are calculated are for predicting missile performance and

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establishing the missile design. Therefore, aerodynamic prediction techniques that satisfy the accuracy requirements associated with these processes are desired. Historically, prediction accuracies have been related directly to the coefficients. For example, a normal force coefficient prediction within 10% of data might be judged as good agreement. But what does this mean in terms of missile range, maneuverability, or wing size? This paper presents the equations that relate performance parameters such as range and design parameters such as wing area to aerodynamic coefficients such as C_A and C_N. When these euations are differentiated with respect to the aerodynamic coefficients and simplifed, the resulting equations relate aerodynamic coefficient accuracy directly to error in performance or design parameters. As a consequence, instead of specifying accuracy by an arbitrary assignment of a coefficient percentage or increment, and allowable error in a performance/design parameter is determined, and the accuracy criteria equation is used to compute the allowable coefficient accuracy. For example, a range error of 10% results in an allowable C_N accuracy of 20%.

The magnitude of the allowable performance/design errors can be selected to represent an level of design detail: conceptual, preliminary, or point design. However, because of

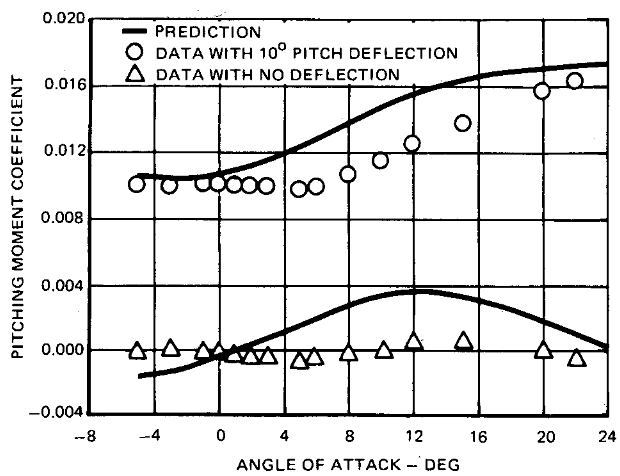

Fig. 1 Is this a good data/theory comparison?

Table 1 Variety of closed-form performance/design relations used

PERFORMANCE	DOMINANT AERO COEFFICIENTS
• POWERED RANGE IN TERMS OF WEIGHT	C_N, C_A
• COAST RANGE TO A GIVEN VELOCITY	C_A
• INSTANTANEOUS TURN RADIUS	C_N OR C_Y
• INSTANTANEOUS TURN RATE	C_N OR C_Y
• INSTANTANEOUS LOAD FACTOR	C_N
• RESPONSE TIME	C_{m_δ}
• CRUISE ALTITUDE AFFECTED BY TRIM	C_m
• SPECIFIC EXCESS POWER	C_{N_α}, C_A
• MANEUVER DECELERATION	
CONFIGURATION DESIGN	
• HORIZONTAL FIN AREA	C_{m_B}
• VERTICAL FIN AREA	C_{n_B}
• DIHEDRAL ANGLE	C_ℓ
• WING AREA	C_{N_B}
• TRIM CONTROL DEFLECTION	C_m
• CENTER OF GRAVITY LOCATION	C_m
AUTOPILOT DESIGN	
• LONGITUDINAL STATIC STABILITY	$C_{m_\alpha}, C_{m_\delta}$
• BANK/ROLL STATIC STABILITY	$C_{\ell_\beta}, C_{n_\beta}, C_{\ell_{\delta A}}$
• YAW STATIC STABILITY	$C_{n_\beta}, C_{\ell_\beta}, C_{n_{\delta R}}$
• ROLL-YAW COUPLING	$C_{n_{\delta A}}, C_{\ell_{\delta R}}, C_{n_{\delta R}}, C_{l_{\delta A}}$

the approximations used in deriving the governing equations, the criteria are best suited for conceptual and preliminary design. In this paper, allowable accuracies are examined for the configurations displayed in Fig. 2. These provide the extremes of aerodynamic characteristics from a conventional to a high-lift aeroconfigured missile concept. The development and application of the accuracy criteria are described in the following paragraphs.

Selection of Governing Equations

Governing equations were selected that relate performance, configuration design, and autopilot design parameters to aerodynamic coefficients. Closed-form equations,[1,2] differential equations of motion,[2] and specific energy expressions[3] were selected for the performance relations. These are often derived with assumptions such as constant velocity or level flight. For the purpose of deriving accuracy criteria, these are not considered restrictive assumptions. These simplified forms emphasize the first-order effect of the aerodynamic coefficients on the performance or design. Table 1 summarizes the performance/design relationships selected to develop the accuracy criteria. The right-hand column indicates the resulting coefficients in each equation. The performance relations relate parameters such as range to aerodynamic coefficients such as normal and axial force. Configuration design relations relate, for example, fin area to body moment coefficients and are typically force and moment balances for configuration components. Autopilot design parameters are related to moment and control levels. Normal and axial force coefficients were substituted for lift and drag coefficients to provide body axis sensitivities.

The selected performance relations are as follows:

Powered range in terms of weight (Ref. 2, Eq. B5.1)

$$R = \frac{2.828}{C_t(\rho S)^{1/2}} \frac{C_L^{1/2}}{C_D} (W_0^{1/2} - W_f^{1/2}) \tag{1}$$

Coast range to a given velocity (Ref. 2, Eqs. B4.8, B1.1)

$$R = \frac{2W}{g\rho S k_{DV}} \left[V_0 - V_f + V_B \ln\left(\frac{V_0 - V_B}{V - V_B}\right)\right] \tag{2}$$

$$k_{DV} = C_{D_0} V^2 / (V - V_B) = \text{const} \tag{3}$$

Instantaneous turn radius (Ref. 3, Eqs. 6-16)

$$r = V^2 / g(n^2 - 1)^{1/2} \tag{4}$$

for bank-to-turn

$$(n^2 - 1)^{1/2} = (C_L qS/W)\sin\phi \tag{5}$$

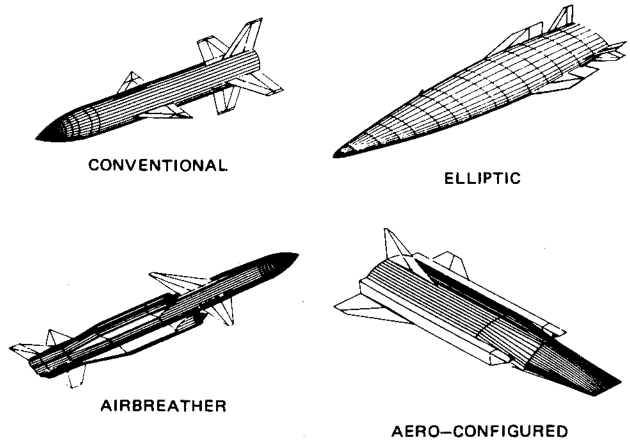

Fig. 2 Accuracy criteria developed for variety of configurations.

for skid-to-turn

$$(n^2-1)^{1/2}=C_YqS/W \tag{6}$$

Instantaneous turn rate (Ref. 3, Eqs. 6-20)

$$\dot{\psi}=\frac{g(n^2-1)^{1/2}}{V} \tag{7}$$

Instantaneous load factor (Ref. 3, Eqs. 6-23)

$$n=\frac{qSC_L}{W} \tag{8}$$

Response time

$$\tau=\sqrt{\frac{2I\alpha}{C_{m_\delta}\delta qSb}} \tag{9}$$

Cruise altitude affected by trim

$$W/qSC_{L_{\text{trim}}}=1 \tag{10}$$

$$C_{L_{\text{trim}}}=C_L+\Delta C_{L_{\text{control}}} \tag{11}$$

$$C_{m_{\text{trim}}}=0=Cm+\frac{(X_{\text{CG}}-X_{\text{CP}_T})}{b}\Delta C_{L_{\text{control}}} \tag{12}$$

Solving for h:

$$h=\frac{1}{\beta_a}\ln\left\{\frac{2W}{\rho_{\text{ref}}V^2S\left[C_L-\dfrac{C_mb}{(X_{\text{CG}}-X_{\text{CP}_T})}\right]}\right\} \tag{13}$$

Specific excess power (Ref. 4, Eq. 5)

$$P_s=\frac{V}{W}\left\{T-\left[C_{D_0}+k\left(\frac{Wn}{qS}\right)^2\right]qS\right\} \tag{14}$$

Maneuver deceleration (Ref. 3, Eqs. 5-31)

$$\dot{V}=-C_DqS/M \tag{15}$$

These relations are not intended to be complete; they contain representative equations that can be used to develop accuracy criteria. The response time [Eq. (9)] has no reference and was developed by determining the time required to pitch a missile through an angle of attack α, using a control deflection δ. Also, Eq. (13) was derived from the force and moment balances [Eqs. (10-12)].

The selected configuration design relations are derived from force and moment balances commonly used in the design of missiles. They include fin and wing sizing, dihedral and trim deflection angle definition, and center-of-gravity location. The equations are as follows:

Horizontal fin area sizing based on pitching moment

$$C_m=0=C_{m_B}+\frac{(X_{\text{CG}}-X_{\text{CP}_T})C'_{N_T}A_T}{bS} \tag{16}$$

Solving for A_T:

$$A_T=\frac{C_{m_B}bS}{(X_{CG}-X_{\text{CP}_T})C'_{N_T}} \tag{17}$$

Verifical fin area sizing based on yawing moment

$$C_n=0=C_{n_B}+\frac{(X_{\text{CG}}-X_{\text{CP}_V})C'_{Y_V}A_V}{bS} \tag{18}$$

Solving for A_V:

$$A_V=\frac{-C_{n_B}bS}{(X_{\text{CG}}-X_{\text{CP}_V})C'_{Y_V}} \tag{19}$$

Dihedral angle based on rolling moment

$$C_l=0=C_{l_B}-\frac{Y_{\text{CP}}}{b}C_{N_{\alpha W}}\beta\sin\Gamma \tag{20}$$

Solving for $\sin\Gamma$:

$$\sin\Gamma=\frac{-C_{l_B}b}{Y_{\text{CP}}C_{N_{\alpha W}}\beta} \tag{21}$$

Wing area sizing based on normal force

$$C_N=C_{N_B}+C'_{N_W}A_W/S \tag{22}$$

Solving for A_W:

$$A_W=\frac{(C_N-C_{N_B})S}{C'_{N_W}} \tag{23}$$

Trim control deflection

$$C_{m_{\text{trim}}}=0=C_m+C_{m_\delta}\delta_{\text{trim}} \tag{24}$$

Solving for δ_{trim}:

$$\delta_{\text{trim}}=-C_m/C_{m_\delta} \tag{25}$$

Center of gravity location

$$C_m=(X_{\text{CG}}-X_{\text{CP}})C_N \tag{26}$$

Solving for X_{CG}:

$$X_{\text{CG}}=\frac{C_m}{C_N}+X_{\text{CP}} \tag{27}$$

The autopilot design relations are as follows:

Static stability

$$K_S=\delta/\alpha=C_{m_\alpha}/C_{m_\delta} \tag{28}$$

Bank/roll static stability

$$K_{S1}=\delta A/\beta=\frac{\alpha}{|\alpha|}\frac{C_{l_\beta}\cos\alpha+C_{n_\beta}\sin\alpha}{C_{l\delta_A}} \tag{29}$$

Yaw stability

$$K_{S3}=\delta R/\beta=-\frac{C_{n_\beta}\cos\alpha-C_{l_\beta}\sin\alpha}{C_{n\delta R}} \tag{30}$$

Roll-yaw control cross coupling (Ref. 5, Fig. 10)

$$K_X=\frac{C_{n_{\delta A}}C_{l_{\delta R}}}{C_{n_{\delta R}}C_{l_{\delta A}}} \tag{31}$$

These equations define the stability parameters K_S, K_{S1}, K_{S3}, and K_X that are used in autopilot design. For example, the static stability parameter K_S is the ratio of $C_{m_\alpha}/C_{m_\delta}$ at any angle of attack. It is a measure of the amount of control deflection required to achieve a given change in angle of attack. For a very stable vehicle, C_{m_q} is large and high control deflections are required. Large control deflections are also required if $C_{m\delta}$ is small. In either case, the vehicle is difficult to control. This is indicated by a large value of K_S. Similar magnitudes arise for K_{S1} and K_{S3}. A feasible autopilot design is possible for these parameters between the values of approximately -0.50 and $+1.0$. The negative limit is for unstable airframes. The cross-coupling parameter K_X is the ratio of the roll-yaw cross-coupling derivatives to the roll-yaw control derivatives. Values of this ratio less than 0.50 are desirable in autopilot designs.[4]

Development of Accuracy Equations

Equations (1-31) were differentiated with respect to aerodynamic coefficients to obtain the accuracy equations. An example of this derivation process beginning with Eq. (8) for instantaneous load factor follows. C_L is assumed approximately equal to C_N and Eq. (8) is differentiated with respect to C_N to give

$$\frac{\partial n}{\partial C_N} = \frac{qS}{W} \tag{32}$$

The right-hand side, qS/W, is then replaced using Eq. (8) and the normalized equation obtained,

$$\frac{\partial n}{n} = \frac{\partial C_N}{C_N} \tag{33}$$

Equation (33) is referred to as the accuracy criteria equation. An allowable error on the performance parameter n is estimated based upon design requirements, e.g.,

$$\frac{\Delta n}{n} = 0.20 \tag{34}$$

Substitution into Eq. (33) provides the allowable accuracy on C_N in the form

$$\frac{\Delta C_N}{C_N} = 0.20 \tag{35}$$

Equation (35) gives the allowable prediction accuracy on C_N which results in a 20% error in load factor.

Table 2 provides the C_N accuracy relations derived from the various performance/design equations (1-27). (All minus signs are dropped because the equations represent absolute values of the errors.) Note that two equation forms result. The simplest form [Eqs. (36-39)] is independent of configuration characteristics or flight conditions. Only the aerodynamic coefficient and performance/design parameter enter the equation. The allowable accuracy is only a function of the allowable error in the performance/design parameter. The second equation form is configuration dependent [Eqs. (40) and (41)]. The accuracy is a function of the performance/design parameters such as weight, reference area, or dynamic pressure and aerodynamic characteristics such as the ratio of wing to body normal force C_{N_W}/C_{N_B}. Therefore each configuration class has a different allowable accuracy.

Table 2 C_N accuracy criteria

PARAMETER	PERFORMANCE/ DESIGN EQUATION	RELATIONSHIP
• POWERED RANGE	(1)	$\frac{\Delta C_N}{C_N} = \frac{2\Delta R}{R}$ (36)
• TURN RADIUS	(4, 5)	$\frac{\Delta C_N}{C_N} = \frac{\Delta r}{r}$ (37)
• TURN RATE	(7)	$\frac{\Delta C_N}{C_N} = \frac{\Delta \dot{\psi}}{\dot{\psi}}$ (38)
• LOAD FACTOR	(8)	$\frac{\Delta C_N}{C_N} = \frac{\Delta n}{n}$ (39)
• SPECIFIC EXCESS POWER	(14)	$\frac{\Delta C_{N_\alpha}}{C_{N_\alpha}^2} = \frac{(q/V)\,\Delta P_S}{(W/S)}$ (40)
• WING SIZE (LOAD FACTOR)	(23)	$\frac{\Delta C_{N_B}}{C_{N_B}} = \frac{C_{N_W}}{C_{N_B}} \frac{\Delta A_W}{A_W}$ (41)

Table 3 presents the C_A accuracy criteria derived from Eqs. (1-27). Equations (42) and (43) are the simple form and Eqs. (44) and (45) have configuration- and flight condition-dependent coefficients. Table 4 summarizes the pitching

Table 3 C_A accuracy criteria

PARAMETER	PERFORMANCE/ DESIGN EQUATION	RELATIONSHIP
• POWERED RANGE	(1)	$\frac{\Delta C_A}{C_A} = \frac{\Delta R}{R}$ (42)
• COAST RANGE	(2, 3)	$\frac{\Delta C_A}{C_A} = \frac{\Delta R}{R}$ (43)
• SPECIFIC EXCESS POWER	(14)	$\Delta C_{A_0} = \frac{W/S}{qV} \Delta P_S$ (44)
• MANEUVER DECELERATION	(15)	$\frac{\Delta C_A}{C_A} = \frac{C_D\,\Delta\dot{V}}{C_A \cos\alpha\,\dot{V}}$ (45)

Table 4 C_m accuracy criteria

PARAMETER	PERFORMANCE/ DESIGN EQUATION	RELATIONSHIP
• RESPONSE TIME	(9)	$\frac{\Delta C_{m_\delta}}{C_{m_\delta}} = \frac{2\Delta\tau}{\tau}$ (46)
• CRUISE ALTITUDE	(10–13)	$\Delta C_m = \left[\frac{\beta_a}{q}\right] \left[\frac{W(X_{CG} - X_{CP_T})}{Sb}\right] \Delta h$ (47)
• HORIZONTAL FIN AREA	(16, 17)	$\frac{\Delta C_{m_B}}{C_{m_B}} = \frac{\Delta A_T}{A_T}$ (48)
• TRIM CONTROL DEFLECTION	(24, 25)	$\frac{\Delta C_m}{C_m} = \frac{\Delta\delta_{TRIM}}{\delta_{TRIM}}$ (49)
• CENTER OF GRAVITY LOCATION	(26, 27)	$\frac{\Delta C_m}{C_m} = \frac{\Delta X_{CG}}{X_{CG} - X_{CP}}$ (50)
• STATIC STABILITY	(28)	$\frac{\Delta C_{m_\delta}}{C_{m_\delta}} = \frac{\Delta K_S}{K_S}$ (51)
• STATIC STABILITY	(28)	$\frac{\Delta C_{m_\alpha}}{C_{m_\alpha}} = \frac{\Delta K_S}{K_S}$ (52)

Table 5 C_Y, C_n accuracy criteria

PARAMETER	PERFORMANCE/ DESIGN EQUATION	RELATIONSHIP
• TURN RADIUS	(4, 6)	$\frac{\Delta C_Y}{C_Y} = \frac{\Delta r}{r}$ (53)
• TURN RATE	(6, 7)	$\frac{\Delta C_Y}{C_Y} = \frac{\Delta\dot{\psi}}{\dot{\psi}}$ (54)
• VERTICAL FIN AREA	(18)	$\frac{\Delta C_{n_B}}{C_{n_B}} = \frac{\Delta A_V}{A_V}$ (55)
• BANK/ROLL STABILITY	(29)	$\Delta C_{n_\beta} = \frac{\lvert\alpha\rvert C_{\ell_{\delta A}}}{\alpha \sin\alpha} \Delta K_{S1}$ (56)
• YAW STABILITY	(30)	$\Delta C_{n_\beta} = C_{n_{\delta R}} \frac{\Delta K_{S3}}{\cos\alpha}$ (57)
• YAW STABILITY	(30)	$\frac{\Delta C_{n_{\delta R}}}{C_{n_{\delta R}}} = \frac{\Delta K_{S3}}{K_{S3}}$ (58)
• CROSS COUPLING	(31)	$\frac{\Delta C_{n_{\delta R}}}{C_{n_{\delta R}}} = \frac{\Delta K_X}{K_X}$ (59)
• CROSS COUPLING	(31)	$\frac{\Delta C_{n_{\delta A}}}{C_{n_{\delta A}}} = \frac{\Delta K_X}{K_X}$ (60)

moment criteria. Care must be taken in applying criteria such as Eqs. (46) and (48) when the moment or its derivative is near zero. For example, if C_m is zero, the response time is infinite. Equations (49) and (50) can be applied by setting a minimum acceptable $\Delta\delta_{\rm trim}$ or ΔX_{CG} such as 2 deg and 0.2 calibers, respectively. This results in the definition of a minimum value for C_m. Tables 5 and 6 present similar criteria for C_Y, C_n, and C_l coefficients.

Table 6 C_l accuracy criteria

PARAMETER	PERFORMANCE/ DESIGN EQUATION	RELATIONSHIP
• DIHEDRAL ANGLE	(20, 21)	$\frac{\Delta C_\ell}{C_\ell} = \frac{\Delta\Gamma}{\Gamma}$ (61)
• BANK/ROLL STATIC STABILITY	(29)	$\Delta C_{\ell_\beta} = \frac{\lvert\alpha\rvert C_{\ell_{\delta A}} \Delta K_{S1}}{\alpha \cos\alpha}$ (62)
• BANK/ROLL STATIC STABILITY	(29)	$\frac{\Delta C_{\ell_{\delta A}}}{C_{\ell_{\delta A}}} = \frac{\Delta K_{S1}}{K_{S1}}$ (63)
• YAW STABILITY	(30)	$\Delta C_{\ell_\beta} = C_{n_{\delta R}} \frac{\Delta K_{S3}}{\sin\alpha}$ (64)
• CROSS COUPLING	(31)	$\frac{\Delta C_{\ell_{\delta A}}}{C_{\ell_{\delta A}}} = \frac{\Delta K_X}{K_X}$ (65)
• CROSS COUPLING	(31)	$\frac{\Delta C_{\ell_{\delta A}}}{C_{\ell_{\delta R}}} = \frac{\Delta K_X}{K_X}$ (66)

Selection of Allowable Performance/Design Errors

Many of the accuracy equations in Tables 2-6 have performance parameter allowable errors (e.g., ΔR) divided by the performance parameters (e.g., R). Therefore, only the fractional error $\Delta R/R$ must be selected. For these cases, Table 7 provides typical allowable errors based upon preliminary design requirements. The user of the accuracy criteria may select other allowable errors based upon his particular design problem. Range is desired within 10%, maneuvering and design parameters within 20%. C_A has the most severe requirement of 10% based upon allowable range accuracies. $C_{m\delta}$ prediction for response time is least severe at 40%.

Allowable errors for autopilot design parameters are more complex because K_S, K_{SI}, K_{S3}, and K_X can have values between 0 and ∞. Tables 8 and 9 describe a recommended approach for determining K. When the parameter K is within acceptable levels for autopilot design, relatively large errors in its magnitude can still result in an acceptable design. Therefore, $K=0.25$ is recommended. At slightly unacceptable levels of K between 1 and 5, large errors are tolerable as long as K is predicted within these levels. This results in an error

Table 7 Many allowable accuracies relate directly to typical allowable errors

PARAMETER	ACCURACY CRITERIA EQUATION	ALLOWABLE ERROR	AERODYNAMIC COEFFICIENT	ALLOWABLE ACCURACY
$\Delta R/R$ (POWERED)	36	0.10	$\Delta C_N/C_N$	0.20
$\Delta r/r$	37	0.20	↓	0.20
$\Delta\dot\psi/\dot\psi$	38	0.20		0.20
$\Delta n/n$	39	0.20		0.20
$\Delta R/R$ (POWERED)	42	0.10	$\Delta C_A/C_A$	0.10
$\Delta R/R$ (COAST)	43	0.10	↓	0.10
$\Delta\tau/\tau$	46	0.20	$\Delta C_{m_\delta}/C_{m_\delta}$	0.40
$\Delta A_T/A_T$	48	0.20	$\Delta C_{m_B}/C_{m_B}$	0.20
$\Delta\delta_{TRIM}/\delta_{TRIM}$	49	0.20	$\Delta C_m/C_m$	0.20
$\Delta X_{CG}/(X_{CG}-X_{CP})$	50	0.20	$\Delta C_m/C_m$	0.20
$\Delta r/r$	53	0.20	$\Delta C_Y/C_Y$	0.20
$\Delta\dot\psi/\dot\psi$	54	0.20		0.20
$\Delta A_Y/A_Y$	55	0.20	$\Delta C_{n_B}/C_{n_B}$	0.20
$\Delta\Gamma/\Gamma$	61	0.20	$\Delta C_\ell/C_\ell$	0.20

Table 8 Typical allowable errors for autopilot design parameters

MAGNITUDE OF K \ ΔK VALUE FOR EQS.	STATIC STABILITY (51, 52)	BANK/ROLL STABILITY (56, 62, 63)	YAW STABILITY (57, 58, 64)
$\lvert K\rvert \leq 1$	0.25	0.25	0.25
$1<\lvert K\rvert \leq 5$	0.25 K	0.25 K	0.25 K
$\lvert K\rvert > 5$	$\Delta C_{m_\delta} = 0.05 \lvert C_{m_\alpha}\rvert$	$\Delta C_{\ell_{\delta A}} = 0.05 \times \lvert C_{\ell_\beta}\cos\alpha + C_{n_\beta}\sin\alpha\rvert$	$\Delta C_{\ell_{\delta R}} = 0.05 \times \lvert C_{n_\beta}\cos\alpha - C_{\ell_\beta}\sin\alpha\rvert$
IF $\frac{\Delta C_{m_\alpha}}{C_{m_\alpha}}, \frac{\Delta C_{\ell_\beta}}{C_{\ell_\beta}}, \frac{\Delta C_{n_\beta}}{C_{n_\beta}} > 0.5$ SET EQUAL TO 0.50			

Table 9 Typical allowable errors for cross-coupling derivatives

ΔK value for Eqs. / Magnitude of K	Cross coupling (59, 60, 65, 66)
$\lvert K\rvert \leq 0.5$	0.125
$0.5 < \lvert K\rvert \leq 2.5$	0.125 K
$\lvert K\rvert > 2.5$	$\Delta C_{n_{\delta R}} = 0.05\, \dfrac{C_{n_{\delta A}} C_{l_{\delta R}}}{C_{l_{\delta A}}}$ $\Delta C_{l_{\delta A}} = 0.05\, \dfrac{C_{n_{\delta A}} C_{l_{\delta R}}}{C_{n_{\delta H}}}$

IF $\dfrac{\Delta C_{n_{\delta A}}}{C_{n_{\delta A}}}, \dfrac{\Delta C_{l_{\delta R}}}{C_{l_{\delta R}}} > 0.50$ SET EQUAL TO 0.50

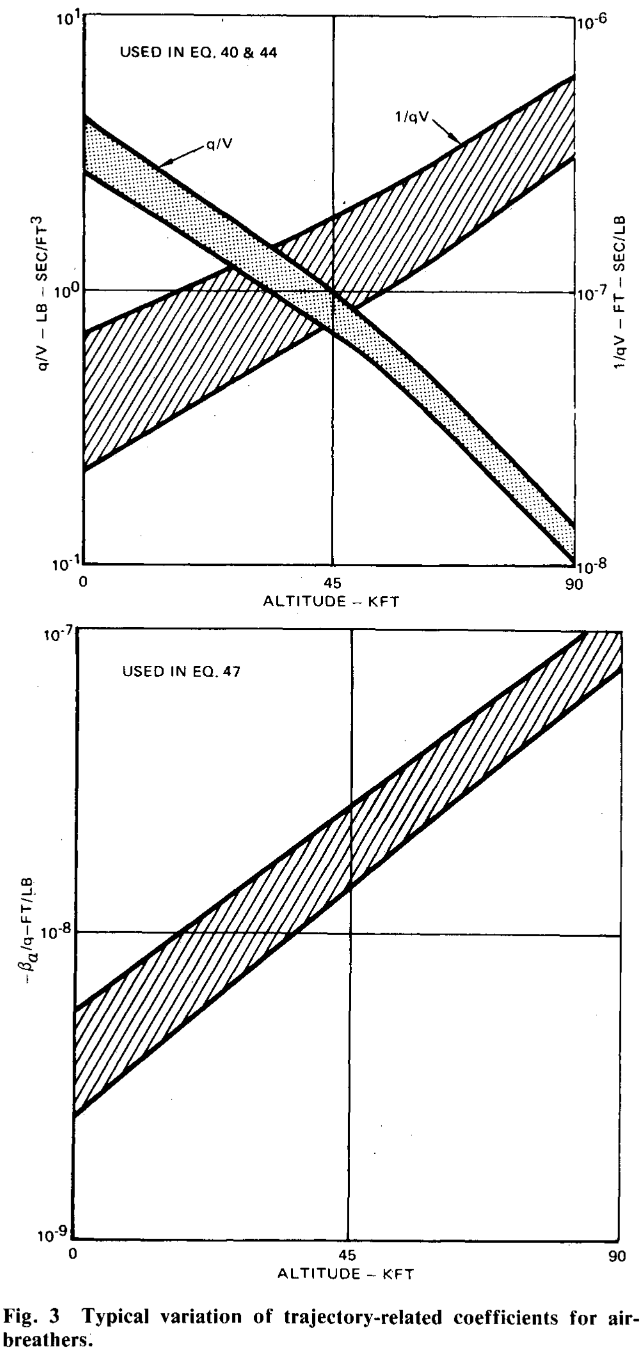

Fig. 3 Typical variation of trajectory-related coefficients for air-breathers.

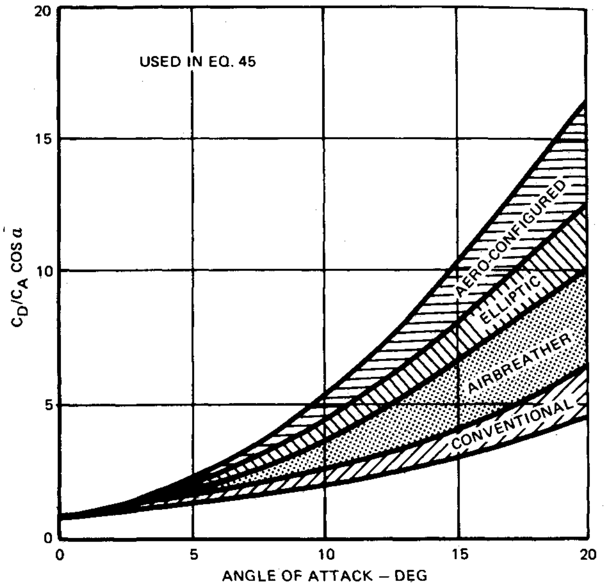

Fig. 4 Comparison of $C_D/C_A\cos\alpha$ for four classes of configurations.

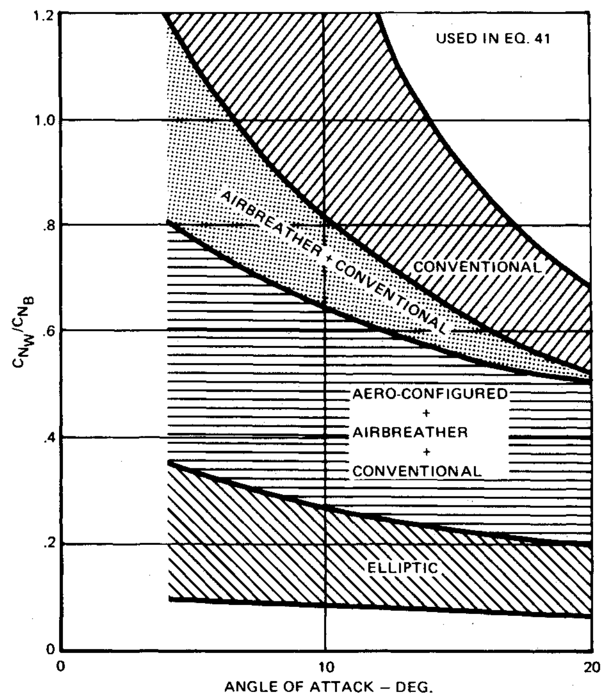

Fig. 5 Comparison of C_{N_W}/C_{N_B} for four classes of configurations.

definition of $\Delta K = 0.25K$. Once K becomes greater than 5, the autopilot design becomes impractical, independent of the level of K. This condition usually occurs when the control derivatives, $C_{m\delta}$, $C_{l_{\delta A}}$, or $C_{n_{\delta R}}$, are close to zero. This often occurs at high angles of attack when the controls are in separated flow regions. For these cases, the expressions shown provide an increment for the control power that is a function of the numerators of the appropriate equations. Table 9 provides a similar approach for definition of the cross-coupling parameter K_X.

Equations (40), (44), and (47) of Tables 2-4 require flight characteristics such as q/V, $1/qV$, and β_a/q. Typical values of these parameters are shown in Fig. 3 as a function of altitude. A ΔP_S of 100 ft/s is approximately 10% of a typical P_S for ramjet missiles of 1000 ft/s. A Δh of 1000 ft represents an acceptable error in estimating cruise altitude. Equation (45) of Table 3 requires the ratio of $C_D/C_A\cos\alpha$. Typical

Table 10 Typical values of configuration/performance dependent coefficients

CONFIGURATION CLASS	COEFFICIENTS FOR ACCURACY CRITERIA EQUATIONS				
	(EQ. 40) (q/V)/(W/S)	(EQ. 41) C_{N_W}/C_{N_B}	(EQ. 44) (W/S)/(qV)	(EQ. 45) $C_D/C_A \cos\alpha$	(EQ. 47) (β_a)(W/Sb)
CONVENTIONAL	(N/A)	0.8	(N/A)	2.5	(N/A)
AIRBREATHER	0.7/(W/S)	0.5	1.8×10^{-7}W/S	3.0	-2.5×10^{-8}W/Sb
ELLIPTIC	(N/A)	0.2	(N/A)	4.0	(N/A)
AERO-CONFIGURED	0.1/(W/S)	0.5	6.3×10^{-7}W/S	5.0	-1.2×10^{-7}W/Sb

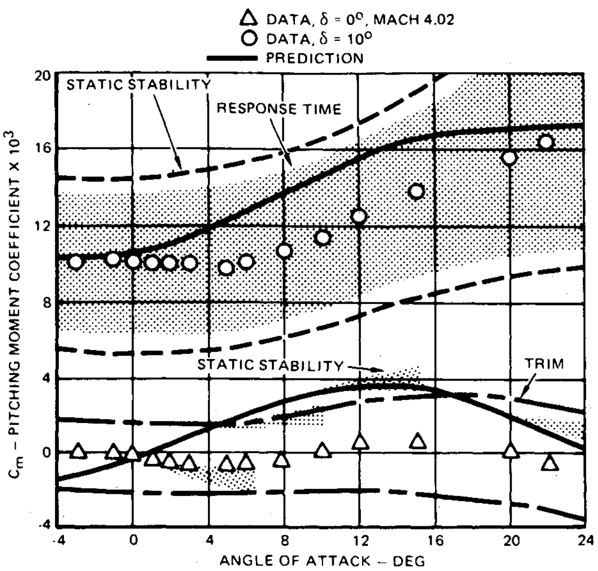

Fig. 6 Application of accuracy criteria to C_m.

values for configurations shown in Fig. 2 are shown in Fig. 4. Wind-tunnel data[5-8] were used to obtain these curves. As the angle of attack increases, the ratio increases, indicating that C_A is a small contribution to C_D. Also, at a fixed angle of attack, the aeroconfigured and elliptic shapes have higher ratios and therefore less important C_A contributions.

Equation (41) of Table 2 requires the ratio of C_{N_W} to C_{N_B}. Figure 5 provides typical variation of this ratio with angle of attack. Since wing size can vary greatly, this ratio is very sensitive to a particular configuration type. Ratios as low as 0.1 and greater than 1.2 are possible. The ratio does decrease as angle of attack increases and body lift becomes more important. Table 10 summarizes typical values of design/performance-dependent coefficients for the four configuration classes and the equations indicated. Note that design characteristics such as W/S and W/Sb appear in the coefficients. Because S and b are reference area and length that vary depending on user preference, they are left to the user to define. Not applicable appears for conventional and elliptic classes because these concepts are assumed to be rocket-powered, boost-glide concepts. Equations (40), (44), and (47) are applicable only to missiles with airbreathing propulsion where P_S and cruise altitude are important design parameters.

Example Coefficient Accuracy

The parameter allowable errors established by Tables 7-10 and Figs. 3-5 were applied to the Mach 4.02 wind-tunnel data of the aeroconfigured noncircular body phase II configuration[8] shown in the lower right of Fig. 2. The predictions were obtained using the Supersonic/Hypersonic Arbitrary Body Program[9] and the ACM rationale[10] that defines the pressure methods to be applied to various regions of the configuration. Figure 6 compares data, predictions, and accuracy bands for pitching moment variation with angle of attack. The triangular and circular symbols are data for 0 and 10 deg pitch deflection, respectively. The two solid lines are predictions for each case. The response time accuracy band is computed by substituting $\Delta\tau/\tau = 0.20$ from Table 7 into Eq. (46) of Table 4 and solving for $\Delta C_{m_\delta} = \pm 0.40\, C_{m_\delta}$. The value of C_{m_δ} used is that given by the test data at each angle of attack. Equations (49), (51), and (52) were used in a similar manner to develop the accuracy bands identified in "trim" and "static stability." The minimum allowable trim deflection error was limited to 2 deg.

The zero deflection prediction is outside the resulting "trim" error band at 4.5-16.5 deg angle of attack and would be a poor prediction for establishing trim deflection within 2 deg. The shaded wedge-shaped regions identified as "static stability" are the slope from Eq. (52) required to accurately predict static stability. At almost all conditions the predicted slope is outside this error band. The prediction with pitch deflection is within the error band established by Eq. (46) for response time and Eq. (51) for static stability. Therefore, the prediction of the effect of deflection on C_m is good for preliminary design purposes.

Conclusions

Accuracy criteria are presented that can be applied at any point in the design process by selecting the appropriate allowable error in performance/design parameters. Criteria are established for the six static force and moment coefficients, C_A, C_N, C_m, C_Y, C_n, and C_ℓ. Allowable errors for performance and design parameters are estimated by the user and the accuracy criteria equations used to relate these to allowable coefficient accuracies. Although examples of allowable errors are given in this paper, the user can select his own to reflect his level of design detail. The criteria developed are best suited for comparing predictions with existing wind-tunnel data during the development and evaluation of prediction techniques.

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